

# Design Of A Flow Sensitive, Context Sensitive Points To Algorithm

Amira Mensi

Centre for Research In Computer  
Department of Mathematics and Systems

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# Plan

- 1 Issue**
- 2 Ultimate Goals**
- 3 General Algorithm**
- 4 Assignment Algorithm**
- 5 Operators**
- 6 Examples**
- 7 Conclusion**

# Issue (1)

```
void foo()
{
    int **z,*v,*w,x=0,y=0,t=0;
    w = &t;
    *z = &y;
    z = &v;
```

```
if(x>0) {
    z = &w;
} else {
    *z = &x;
}
```

## Issue (2)

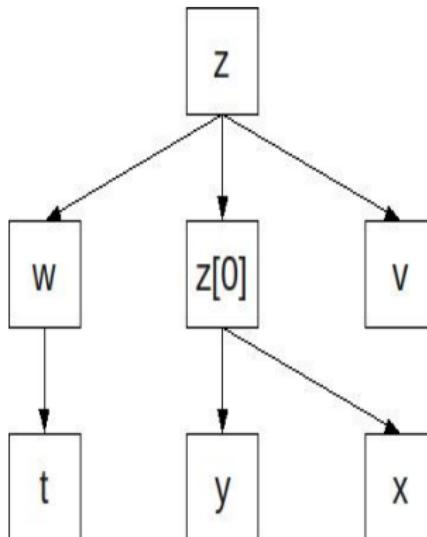


Figure: Memory state

## Issue (3)

Many cases should be taken into account  
[Emami 93]

- $p = \&i;$
- $p = q;$
- $\text{my\_str-} > p = \&i;$
- $\text{my\_str.p} = \&i;$
- $p = \&\text{tab}[i];$
- $\text{foo}(x)- > p = q;$
- ...

Reduce complexity to 2 basic cases

- $p = \&i; //\{(p,i,E)\}$
  - $p = q;$   
 $//\{(q,j,M),(p,j,M)\}$
- pt=(source,sink,approximation)

# Goals

- Design a flow-sensitive algorithm (statement's order is taken into account)
- Design a context-sensitive algorithm (store informations about the call site)  
*[not addressed in this talk]*
- Handle all C features

## Statement Level

A statement can be a:

- sequence
- test
- loop
- whileloop
- goto
- forloop
- expression

|                          |                  |
|--------------------------|------------------|
| Issue                    | Statement Level  |
| Ultimate Goals           | Sequence Level   |
| <b>General Algorithm</b> | Test Level       |
| Assignment Algorithm     | Loop Level       |
| Operators                | Expression Level |
| Examples                 |                  |
| Conclusion               |                  |

## Sequence Level

$$\mathcal{PT} : (i, \sigma) \mapsto (i', \sigma) \quad (1)$$

For  $\llbracket S_1; S_2 \rrbracket$ :

$$\mathcal{PT}\llbracket S_1; S_2 \rrbracket(i, \sigma) = \mathcal{PT}\llbracket S_2 \rrbracket(\mathcal{PT}\llbracket S_1 \rrbracket(i, \sigma)) \quad (2)$$

|                          |                   |
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## Test Level (1)

Exact equation with condition evaluation's for  
 $\llbracket \text{if } E \text{ then } S_1 \text{ else } S_2 \rrbracket$ :

$$\begin{aligned} \mathcal{PT}\llbracket \text{if } E \text{ then } S_1 \text{ else } S_2 \rrbracket(i, \sigma) = & \mathcal{PT}\llbracket S_1 \rrbracket(\mathcal{PT}\llbracket E \rrbracket(i, \sigma)) \wedge \mathcal{E}\llbracket E \rrbracket\sigma \\ & \bigcup \mathcal{PT}\llbracket S_2 \rrbracket(\mathcal{PT}\llbracket \neg E \rrbracket(i, \sigma)) \wedge \neg \mathcal{E}\llbracket E \rrbracket\sigma \end{aligned}$$

|                          |                   |
|--------------------------|-------------------|
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| Operators                | Expression Level  |
| Examples                 |                   |
| Conclusion               |                   |

## Test Level (2)

Approximate equations without condition evaluation's for  
 $\llbracket \text{if } E \text{ then } S_1 \text{ else } S_2 \rrbracket$ :

$$\overline{\mathcal{PT}}\llbracket \text{if } E \text{ then } S_1 \text{ else } S_2 \rrbracket(i, \sigma) = \mathcal{PT}\llbracket S_1 \rrbracket(\mathcal{PT}\llbracket E \rrbracket(i, \sigma)) \\ \bigcup \mathcal{PT}\llbracket S_2 \rrbracket(\mathcal{PT}\llbracket \neg E \rrbracket(i, \sigma))$$

$$\underline{\mathcal{PT}}\llbracket \text{if } E \text{ then } S_1 \text{ else } S_2 \rrbracket(i, \sigma) = \mathcal{PT}\llbracket S_1 \rrbracket(\mathcal{PT}\llbracket E \rrbracket(i, \sigma)) \\ \bigcap \mathcal{PT}\llbracket S_2 \rrbracket(\mathcal{PT}\llbracket \neg E \rrbracket(i, \sigma))$$

with

$$\mathcal{PT}\llbracket \neg E \rrbracket(i, \sigma) = \mathcal{PT}\llbracket E \rrbracket(i, \sigma)$$

|                          |                   |
|--------------------------|-------------------|
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## Loop Level (1)

The exact equation with condition evaluation's for  $\llbracket \text{while } C \text{ do } b \rrbracket$ :

$$\begin{aligned}
\mathcal{PT}[\llbracket \text{while } C \text{ do } b \rrbracket](i, \sigma) &= \mathcal{PT}[\llbracket \text{if } C \text{ then } b; \text{while } C \text{ do } b \rrbracket](i, \sigma) \\
&= \mathcal{PT}[\llbracket b; \text{while } C \text{ do } b \rrbracket](\mathcal{PT}[\llbracket C \rrbracket](i, \sigma)) \wedge \mathcal{E}[\llbracket C \rrbracket]\sigma \\
&\quad \bigcup \mathcal{PT}[\llbracket C \rrbracket](i, \sigma) \wedge \neg \mathcal{E}[\llbracket C \rrbracket]\sigma \\
&= \mathcal{PT}[\llbracket \text{while } C \text{ do } b \rrbracket](\mathcal{PT}[\llbracket b \rrbracket](\mathcal{PT}[\llbracket C \rrbracket](i, \sigma))) \wedge \mathcal{E}[\llbracket C \rrbracket]\sigma \\
&\quad \bigcup \mathcal{PT}[\llbracket C \rrbracket](i, \sigma) \wedge \neg \mathcal{E}[\llbracket C \rrbracket]\sigma
\end{aligned}$$

|                          |                   |
|--------------------------|-------------------|
| Issue                    | Statement Level   |
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| Operators                | Expression Level  |
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## Loop Level (2)

The approximate equations for  $\llbracket \text{while } C \text{ do } b \rrbracket$ , no condition evaluation's is taken into account:

$$\begin{aligned}
 \overline{\mathcal{PT}}\llbracket \text{while } C \text{ do } b \rrbracket(i, \sigma) &= \overline{\mathcal{PT}}\llbracket \text{if } C \text{ then } b; \text{while } C \text{ do } b \rrbracket(i, \sigma) \\
 &= \overline{\mathcal{PT}}\llbracket b; \text{while } C \text{ do } b \rrbracket(\overline{\mathcal{PT}}\llbracket C \rrbracket(i, \sigma)) \bigcup \overline{\mathcal{PT}}\llbracket C \rrbracket(i, \sigma) \\
 &= \overline{\mathcal{PT}}\llbracket \text{while } C \text{ do } b \rrbracket(\overline{\mathcal{PT}}\llbracket b \rrbracket(\overline{\mathcal{PT}}\llbracket C \rrbracket(i, \sigma))) \\
 &\quad \bigcup \overline{\mathcal{PT}}\llbracket C \rrbracket(i, \sigma)
 \end{aligned}$$

$$\begin{aligned}
 \underline{\mathcal{PT}}\llbracket \text{while } C \text{ do } b \rrbracket(i, \sigma) &= \underline{\mathcal{PT}}\llbracket \text{if } C \text{ then } b; \text{while } C \text{ do } b \rrbracket(i, \sigma) \\
 &= \underline{\mathcal{PT}}\llbracket b; \text{while } C \text{ do } b \rrbracket(\underline{\mathcal{PT}}\llbracket C \rrbracket(i, \sigma)) \bigcap \underline{\mathcal{PT}}\llbracket C \rrbracket(i, \sigma) \\
 &= \underline{\mathcal{PT}}\llbracket \text{while } C \text{ do } b \rrbracket(\underline{\mathcal{PT}}\llbracket b \rrbracket(\underline{\mathcal{PT}}\llbracket C \rrbracket(i, \sigma))) \\
 &\quad \bigcap \underline{\mathcal{PT}}\llbracket C \rrbracket(i, \sigma)
 \end{aligned}$$

## Loop Level (3)

- $\mathcal{PT}[\![\text{while } C \text{ do } b]\!]$  replaced by  $\mathcal{W}$ ;
- $\mathcal{PT}[\![b]\!]$  replaced by  $\mathcal{B}$ ;
- $\mathcal{PT}[\![C]\!]$  replaced by  $\mathcal{C}$ ;

To obtain a recurrence equation

more precisely:

$$\mathcal{W}(i, \sigma) = (\mathcal{W} \circ \mathcal{B} \circ \mathcal{C})(i, \sigma) \cup \mathcal{C}(i, \sigma)$$

$$\mathcal{W}(i, \sigma) = ((\mathcal{W} \circ \mathcal{B} \circ \mathcal{C}) \cup \mathcal{C}) \circ \mathcal{B} \circ \mathcal{C}(i, \sigma) \cup \mathcal{C}(i, \sigma)$$

$$= \bigcup_{k=1}^{\infty} (\mathcal{B} \circ \mathcal{C})^{k-1} \circ \mathcal{C}$$

$$Y_0 = \mathcal{C}$$

$$Y_i = Y_{i-1} \cup \mathcal{B} \circ \mathcal{C} \circ Y_{i-1}$$

## Expression Level

An expression could be a :

- reference
- range
- call
- cast
- sizeofexpression
- subscript
- application

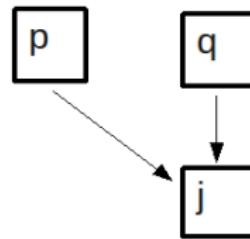
- At the IR an assignment is a call so a sub case of expression
- The assignment operator generates *points to* relations
- For the other sub cases a recursive descent is performed

# Assignment Algorithm (1)

$p = \&i;$



$p = q;$



## Assignment Algorithm (2)

`lhs = rhs; // in is the set of initial points to`

- 1** Compute side effects of lhs and rhs
- 2** Extract may/must relations
- 3** Evaluate lhs using *points to* relations
- 4** switch rhs type's :
  - if rhs is an lhs => Evaluate rhs using *points to* relations
  - else change rhs into an lhs => Evaluate rhs using *points to* relations
  - for the other sub cases a recursive descent is performed
- 5** compute must/may kill set
- 6** compute must/may gen set
- 7** compute must/may out set
- 8**  $\text{out} = \text{out\_may} \sqcup \text{out\_must}$

## Normalization of lhs with memory access paths

- 1** translate  $**p$  into  $p[0][0]$ ,  $my\_str.p$  into  $my\_str[.q]$ ,  $my\_str \rightarrow q$  en  $my\_str[0][.q]$
- 2** evaluate constant path using *points-to*  $p[0]$  replaced by  $i$  if  $(p,i,E)$

# The kill operator

- $constant - path : CP = Module \times Name \times Type \times Vref$
- $points - to : PT = CP \times CP \times A$
- $L = \{(m_l, n_l, t_l, vref_l)\}$

$$Kill_{MUST}(L, In) = \{(s, d, a) \in In \mid \forall l \in L \wedge opkillmust_{cp}(s, l) \wedge |L| = 1\}$$

$$Kill_{MAY}(L, In) = \{(s, d, a) \in In \mid \exists l \in L \wedge opkillmay_{cp}(s, l)\}$$

## Examples (1): Program

```
typedef struct LinkedList{
    int *val;
    struct LinkedList *next;
}list;

list* initialize()
{
    int *pi;
```

```
list *l=NULL, *nl;
pi = malloc(sizeof(int));
nl = malloc(sizeof(list*));
nl->val = pi;

return l;
}
```

## Examples (1): Result

```
list * initialize()
{
// points to = {}
int *pi;
// points to = {}
list *l = (void *) 0, *nl;
// points to = {}

pi = malloc(sizeof(int));
// {(pi,*HEAP*_l_15,-Exact-)}
```

```
nl = malloc(sizeof(list *));
// {nl,*HEAP*_l_16,-Exact-};(pi,*HEAP*
// _l_15,-Exact-)
nl->val = pi;
// {(*HEAP*_l_16[val],*HEAP*_l_15,-Exact-)
// ;(nl,*HEAP*_l_16,
// // -Exact-);(pi,*HEAP*_l_15,-Exact-)}
}

return l;
}
```

- heap objects designated by `*heap*NumberStatement`
- dereferencing pointers evaluated using computed *points-to*

## Examples (2): Program

```
typedef struct ListeChaine{  
    int val;  
    struct ListeChaine * next;  
}liste;  
int count(liste* p)  
{  
    liste* q = p;
```

```
    int i = 0;  
    while(p != NULL){  
        i++;  
        p = p->next;  
    }  
    return i;  
}
```

- while loop: traverse a linked list
- while loop changed into consecutive if (experimentally)
- dereferencing pointers evaluated using computed *points-to*

## Examples (2): Result

```
typedef struct ListeChaine{  
    int val;  
    struct ListeChaine * next;  
}liste;  
// points-to {(p,p_formal,Exact)}  
int count(liste* p)  
{// points-to {(p,p_formal,Exact)}  
    liste* q = p;  
    // points-to {(p,p_formal,Exact);(q,  
        p_formal,Exact)}  
    int i = 0;  
    // points-to {(p,p_formal,Exact);(q,  
        p_formal,Exact)}  
    if(p != NULL){  
        // points-to {(p,p_formal,Exact);(q,  
            p_formal,Exact)}  
        i++;  
        // points-to {(p,p_formal,Exact);(q,  
            p_formal,Exact)}
```

```
    p = p->next;  
    // points-to {(p,p_formal[next],Exact);(q,  
        p_formal,Exact)}  
}  
    if(p != NULL){  
        // points-to {(p,p_formal[next],May);(q,  
            p_formal,Exact)}  
        i++;  
        // points-to {(p,p_formal[next],May);(q,  
            p_formal,Exact)}  
        p = p->next;  
        // points-to {(p,p_formal[next],May);(p,  
            p_formal[next][next],May);(q,  
            p_formal,Exact)}  
    }  
    return i;  
}
```

## Examples (2): Result

- sink increases over iterations
- appearance of a common prefix

## Examples (3): Program

```
typedef struct ListeChaine{  
    int* val;  
    struct ListeChaine * next;  
}liste;  
int count(liste* p)  
{  
    liste* q = p;  
    int i = 0, j;  
    int tab[20];  
    for(j=0; j<20; j++){
```

```
        tab[j] = j  
    }  
    while(p != NULL){  
        p->val = &tab[i];  
        p = p->next;  
        i++;  
    }  
    return i;  
}
```

## Examples (3): Result

$i = 0 : Y_0 = \{(q, p\_formal, E), (p, p\_formal, E)\}$

$i = 1 : Y_1 = \{(q, p\_formal, E), (p\_formal[val], tab[*], M), (p, p\_formal[next], M)\}$

$i = 2 : Y_2 = \{(q, p\_formal, E), (p\_formal[val], tab[*], M), (p, p\_formal[next], M)\}$

$\quad \cup \{(q, p\_formal, E), (p\_formal[next][val], tab[*], E),$

$\quad (p, p\_formal[next][next], E)\}$

$= \{(q, p\_formal, E), (p\_formal[val], tab[*], M), (p, p\_formal[next], M);$

$\quad (p, p\_formal[next][next], M), (p\_formal[next][val], tab[*], M)\}$

$i = n : Y_n = \{(q, p\_formal, E), (p\_formal[val], tab[*], M), (p, p\_formal[next]^n, M),$

$\quad (p\_formal[next]^n[val], tab[*], M), (p, p\_formal[next]^{n-1}, M),$

$\quad (p\_formal[next]^{n-1}[val], tab[*], M), (p, p\_formal[next]^{n-2}, M), \dots\}$

## Examples (3): Result

- sink increases over iterations
- source increases over iterations
- appearance of a common prefix
- appearance of a new prefix = common\_prefix[val]

## Examples (4): Program

```
typedef struct LinkedList{
    int *val;
    struct LinkedList *next;
}list;

list* initialize()
{
    int *pi, i;
    list *l=NULL, *nl;

    if(!feof(stdin)){
        scanf("%d",&i);
        pi = malloc(sizeof(int));
        *pi = i;
        nl = malloc(sizeof(list*));
    }
}
```

```
nl->val = pi;
nl->next = l;
l = nl;
if(!feof(stdin)){
    scanf("%d",&i);
    pi = malloc(sizeof(int));
    *pi = i;
    nl = malloc(sizeof(list*));
    nl->val = pi;
    nl->next = l;
    l = nl;
}
}
return l;
}
```

## Examples (4): Result

```

if (!feof(stdin)) {
  pi = malloc(sizeof(int));
  //{{(pi,*HEAP*_l_17,E)}
  nl = malloc(sizeof(list *));
  //{{(nl,*HEAP*_l_19,E);(pi,*HEAP*_l_17,E)}
  nl->val = pi;
  //{{(*HEAP*_l_19[val],*HEAP*_l_17,E);(nl,*HEAP*_l_19,E);(pi,*HEAP*_l_17,E)}
  nl->next = l;
  //{{(*HEAP*_l_19[val],*HEAP*_l_17,E);(nl,*HEAP*_l_19,E);(pi,*HEAP*_l_17,E)}
  l = nl;
  //{{(*HEAP*_l_19[val],*HEAP*_l_17,E);(l,*HEAP*_l_19,E);(nl,*HEAP*_l_19,E);
  //{{(pi,*HEAP*_l_17,E)}
  if (!feof(stdin)) {
    //{{(*HEAP*_l_19[val],*HEAP*_l_17,M);(l,*HEAP*_l_19,M);(nl,*HEAP*_l_19,M);
    //{{(pi,*HEAP*_l_17,M)}
    pi = malloc(sizeof(int));
  }
}

```

```

//{{(*HEAP*_l_19[val],*HEAP*_l_17,M);(l,*HEAP*_l_19,M);(nl,*HEAP*_l_19,M);
//{{(pi,*HEAP*_l_25,E)}
nl = malloc(sizeof(list *));
//{{(*HEAP*_l_19[val],*HEAP*_l_17,M);(l,*HEAP*_l_19,M);(nl,*HEAP*_l_27,E);
//{{(pi,*HEAP*_l_25,E)}
nl->val = pi;
//{{(*HEAP*_l_27[val],*HEAP*_l_25,E);
//{{(l,*HEAP*_l_19,M);(nl,*HEAP*_l_27,E);(pi,*HEAP*_l_25,E)}
nl->next = l;
//{{(*HEAP*_l_27[next],*HEAP*_l_19,E);
//{{(*HEAP*_l_27[val],*HEAP*_l_25,E);(l,*HEAP*_l_19,M);(nl,*HEAP*_l_27,E);(pi,*HEAP*_l_25,E)}
l = nl;
}
}

```

## Examples (4): Result

- source/sink increases over iterations
  - more information about heap sites
  - the number of *points to* relations increases
  - need to find a fix point ?
- 1** k-limiting: define a length limit
- 2** use lattice order:

$$\{(q, pn^i, a)\} \cup in = \{(q, anywhere, a)\} \cup in \text{ si } i \geq k \quad (3)$$

# Prospects

- handle more C features
- interprocedual analysis using *points to* summaries